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Quantum dynamical phase transition in a system with many-body interactions

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Abstract

Recent experiments, [G.A. Álvarez, E.P. Danieli, P.R. Levstein, H.M. Pastawski, J. Chem. Phys. 124 (2006) 194507], have reported the observation of a quantum dynamical phase transition in the dynamics of a spin swapping gate. In order to explain this result from a microscopic perspective, we introduce a Hamiltonian model of a two level system with many-body interactions with an environment whose excitation dynamics is fully solved within the Keldysh formalism. If a particle starts in one of the states of the isolated system, the return probability oscillates with the Rabi frequency ω_0. For weak interactions with the environment $1/\tau_{SE} < 2\omega_0$, we find a slower oscillation whose amplitude decays with a rate $1/\tau_\phi = 1/(2\tau_{SE})$. However, beyond a finite critical interaction with the environment, $1/\tau_{SE} > 2\omega_0$, the decay rate becomes $1/\tau_\phi \propto \omega_0^2\tau_{SE}$. The oscillation period diverges showing a quantum dynamical phase transition to a Quantum Zeno phase consistent with the experimental observations.

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Ideal quantum information processing (QIP) is the evolution of a system under a controlled Hamiltonian. However, the environment [1] can perturb that dynamics, smoothly degrading the quantum interferences within a "decoherence" rate, $1/\tau_\phi$. Although one expects $1/\tau_\phi$ to be proportional to the system–environment (SE) interaction rate $1/\tau_{SE}$, there are conditions where $1/\tau_\phi$ does not depend on it [2]. This phenomenon was interpreted [3] as the onset of a Lyapunov phase, where the decay rate is the Lyapunov exponent λ characterizing the complexity of the classical system. The description of this transition, $1/\tau_\phi = \min[1/\tau_{SE}, \lambda]$, requires evaluation of the observables beyond perturbation theory [3]. In a recent NMR experiment on a swapping gate [4], another surprising dynamical transition was observed. There, a two spin system oscillates between two equivalent states, $A = \uparrow\downarrow$ and $B = \downarrow\uparrow$, at a Rabi frequency $\omega_0 = 2V_{AB}/\hbar$ when the coupling V_{AB} is turned on. Weak interactions with the environment ($1/\tau_{SE} < 2\omega_0$) produce slightly slower oscillations which decay at a rate $1/\tau_\phi = 1/(2\tau_{SE})$. However, both observables, the swapping frequency ω and $1/\tau_\phi$ depend *non-analytically* on $1/\tau_{SE}$. At a critical strength $1/\tau_{SE}^c = 2\omega_0$, the oscillation freezes indicating a *transition* to a different *dynamical regime*. The initial state now decays to equilibrium at a slow rate $1/\tau_\phi \propto \omega_0^2\tau_{SE}$ which cancels for strong SE interaction. This last regime was interpreted as a Quantum Zeno phase, where the dynamics is inhibited by frequent "observations" [5] by the environment at the phenomenological rate $1/\tau_{SE}$. Indeed, a quantum freeze can arise as a pure dynamical process governed by strictly unitary evolutions [6,7]. A similar dynamical transition is expected for the spin-boson model where the bath is described through its spectral density $J(\omega) \sim \alpha\omega^s$ [8]. For an Ohmic bath ($s = 1$) it is known to have at least two dynamical transitions as a function of α, the system–environment interaction strength. For $\alpha < 1/2$ it presents damped oscillations. The overdamped regime of incoherent relaxation sets up at $\frac{1}{2} < \alpha < 1$, while at $\alpha > 1$ it undergoes a transition into a totally localized phase. However, it is less clear how the system behaves when the bath is in the sub-Ohmic regime ($s < 1$), and much work is devoted to this issue [9,10]. This is because fermionic environments at

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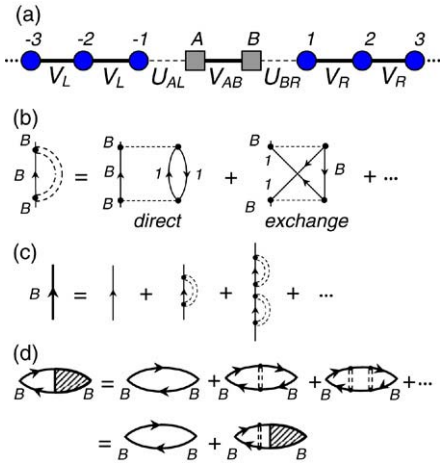

Fig. 1. (Color online) (a) Squares (circles) represent system (environment) states. Solid lines are tunneling matrix elements while dashed lines are through-space Coulomb interactions. (b) The relevant self-energy diagrams in a local basis. Lines are exact Green's functions of the system and the environment in absence of Coulomb interactions (dashed lines). (c) Retarded Green's function of the system–environment at site B accounting for the Coulomb interaction through the self-energy to infinite order. (d) Particle density at site B. The dashed lines are local interactions in time and space.

very low temperature correspond to this limit and the traditional theoretical approach [8] does not hold. However, this regime does not describe our experimental conditions of very high temperature and $s \approx 0$. Here, we consider a two level system coupled to a fermionic environment whose dynamics is much faster than that of the system. Within the high temperature regime, this describes the real experimental situation [4]. Since the spin system has a natural mapping into fermions, we will resort to the Keldysh non-equilibrium formalism [11] to obtain a microscopic derivation of the system–environment interaction rate $1/\tau_{\rm SE}$ and to evaluate the observables. The result relies on the limit of infinite degrees of freedom in the environment. This limit is responsible for the non-analytic behavior of the observables characterizing the quantum dynamical phase transition [4].

We consider a "system" with a *single* electron occupying one of two coupled states, A or B, each interacting with a corresponding electron reservoir (the "environment"). The total system, represented in Fig. 1(a), has the Hamiltonian $\widehat{\mathcal{H}} = \widehat{\mathcal{H}}_{\rm S} + \widehat{\mathcal{H}}_{\rm E} + \widehat{\mathcal{H}}_{\rm SE}$ where the first term is

$$\widehat{\mathcal{H}}_{\rm S} = E_A \hat{c}_A^+ \hat{c}_A + E_B \hat{c}_B^+ \hat{c}_B - V_{AB}\left(\hat{c}_A^+ \hat{c}_B + \hat{c}_B^+ \hat{c}_A\right). \tag{1}$$

Here, $\hat{c}_i^+(\hat{c}_i)$ are the standard fermionic creation (destruction) operators. E_i stands for the energy of the i-th local state whose spin index is omitted. V_{AB} yields the natural frequency, $\omega_0 = 2V_{AB}/\hbar$. The environment is

$$\widehat{\mathcal{H}}_{\rm E} = \sum_{i=-\infty(i\neq 0)}^{\infty}\left(E_{i/|i|}\hat{c}_i^+\hat{c}_i - V_{i/|i|}\left(\hat{c}_i^+\hat{c}_{i+i/|i|} + \hat{c}_{i+i/|i|}^+\hat{c}_i\right)\right) \tag{2}$$

where the sums on negative (positive) index describe a semi-infinite chain to the left (right) acting as a reservoir. $E_{-1} \equiv E_L$ and $E_1 \equiv E_R$ are site energies while $V_{-1} \equiv V_L$ and $V_1 \equiv V_R$ are adjacent site hoppings. In Ref. [4], only one lateral chain attached to the system was used to model the experimental condition. Here, we use a double reservoir to show that the transition could not be seen in the single particle green function, but the observable density must be evaluated as will be clear below. The system–environment interaction is modeled with a through-space interaction

$$\begin{aligned}\widehat{\mathcal{H}}_{\rm SE} = \sum_{\alpha=\uparrow,\downarrow}\Bigg\{&\sum_{\beta=\uparrow,\downarrow} U_{B\rm R}^{(\rm dir.)}\hat{c}_{B\beta}^+\hat{c}_{B\beta}\hat{c}_{1\alpha}^+\hat{c}_{1\alpha} \\ &+ U_{B\rm R}^{(\rm exch.)}\hat{c}_{B\alpha}^+\hat{c}_{1\alpha}\hat{c}_{1\alpha}^+\hat{c}_{B\alpha} + \sum_{\beta=\uparrow,\downarrow} U_{A\rm L}^{(\rm dir.)}\,\hat{c}_{A\beta}^+\hat{c}_{A\beta}\hat{c}_{-1\alpha}^+\hat{c}_{-1\alpha} \\ &+ U_{A\rm L}^{(\rm exch.)}\hat{c}_{A\alpha}^+\hat{c}_{-1\alpha}\hat{c}_{-1\alpha}^+\hat{c}_{A\alpha}\Bigg\}.\end{aligned} \tag{3}$$

The first two terms within the curly brackets represent the Coulomb interaction of an electron in state B with an electron in the first site of the reservoir to the right. $U_{B\rm R}^{(\rm dir.)}$ is the standard direct integral and $U_{B\rm R}^{(\rm exch.)}$ is the exchange one. Analogously, the third and fourth terms are the interaction with the reservoir to the left.

A complete norm preserving solution requires the evaluation of the reduced particle and hole density functions $G_{ij}^<(t_2,t_1) = \frac{\rm i}{\hbar}\langle\Psi|\hat{c}_j^+(t_1)\hat{c}_i(t_2)|\Psi\rangle$ and $G_{ij}^>(t_2,t_1) = -\frac{\rm i}{\hbar}\langle\Psi|\hat{c}_i(t_2)\hat{c}_j^+(t_1)|\Psi\rangle$ that describe temporal and spacial correlations. Here, the creation and destruction operators are in the Heisenberg representation and $|\Psi\rangle = \hat{c}_A^+\hat{c}_B|\Psi_0\rangle$ is an initial non-equilibrium many-body state built from the non-interacting equilibrium state $|\Psi_0\rangle$. The retarded Green's function $G_{ij}^{\rm R}(t_2,t_1) = [G_{ji}^{\rm A}(t_1,t_2)]^\dagger = \theta(t_2,t_1)[G_{ij}^>(t_2,t_1) - G_{ij}^<(t_2,t_1)]$ describes the probability amplitude of finding an electron at site i after placing it at site j and letting it evolve under the total Hamiltonian for a time $t_2 - t_1$. By restricting the analysis to $i,j \in \{A,B\}$, $\mathbf{G}^<(t,t)$ is the single particle 2×2 density matrix and $\mathbf{G}^{\rm R}(t_2,t_1)$ is an effective evolution operator in this reduced space. In absence of SE interaction, the Green's function is easily evaluated in its energy representation $\mathbf{G}^{0\rm R}(\varepsilon) = \int \mathbf{G}^{0\rm R}(t)\exp[{\rm i}\varepsilon t/\hbar]\,{\rm d}t = [\varepsilon\mathbf{I} - \mathbf{H}_{\rm S}]^{-1}$. Conversely, the interacting Green's function defines the reduced effective Hamiltonian and the self-energies $\mathbf{\Sigma}^{\rm R}(\varepsilon)$ [12], $\mathbf{H}_{\rm eff.}(\varepsilon) \equiv \varepsilon\mathbf{I} - [\mathbf{G}^{\rm R}(\varepsilon)]^{-1} = \mathbf{H}_{\rm S} + \mathbf{\Sigma}^{\rm R}(\varepsilon)$, where the exact perturbed dynamics is contained in the nonlinear dependence of the self-energies $\Sigma^{\rm R}$ on ε. For infinite reservoirs ${\rm Re}\,\Sigma^{\rm R}(\varepsilon_\nu^o)$ represents the "shift" of the system's eigen-energies ε_ν^o and $-{\rm Im}\,\Sigma^{\rm R}(\varepsilon_\nu^o)/\hbar = 1/(2\tau_{\rm SE})$ accounts for their "decay rate" into collective SE eigenstates in agreement with a self-consistent Fermi Golden Rule (FGR) [13], i.e. the evolution with $\mathbf{H}_{\rm eff.}$ is non-unitary.

The complete dynamics will be obtained by resorting to the Keldysh formalism [11]. This allows the evaluation of the relevant density–density correlations within a norm conserving scheme [11]:

$$\begin{aligned}\mathbf{G}^<(t_2,t_1) &= \hbar^2\,\mathbf{G}^{\rm R}(t_2,0)\,\mathbf{G}^<(0,0)\,\mathbf{G}^{\rm A}(0,t_1) \\ &+ \int_0^{t_2}\int_0^{t_1}{\rm d}t_k{\rm d}t_l\mathbf{G}^{\rm R}(t_2,t_k)\,\mathbf{\Sigma}^<(t_k,t_l)\,\mathbf{G}^{\rm A}(t_l,t_1).\end{aligned} \tag{4}$$

The first term in the right hand side stands for the "coherent" evolution while second term contains "incoherent reinjections", described by the injection self-energy, $\mathbf{\Sigma}^{<}$, that compensates any leak from the coherent evolution [14]. Solving Eq. (4) requires the particle (hole) injection and retarded self-energies, $\Sigma^{<(>)}(t_2,t_1)$ and $\Sigma^{\mathrm{R}}(t_2,t_1)=\theta(t_1,t_2)[\Sigma^{>}(t_2,t_1)-\Sigma^{<}(t_2,t_1)]$. For this, we use a perturbative expansion on $\widehat{\mathcal{H}}_{\mathrm{SE}}$. The first order gives the standard Hartree-Fock energy corrections which, being real, do not contribute to $\Sigma^{<}$. The second order terms [11], sketched in Fig. 1(b), contribute to $\Sigma^{\gtrless}$, and in the space representation:

$$\frac{\Sigma^{\lessgtr}_{ij}(t_k,t_l)}{\hbar^2}=|U_{is}|^2\,G^{\lessgtr}_{ss}(t_k,t_l)\,G^{\gtrless}_{ss}(t_l,t_k)\,G^{\lessgtr}_{ii}(t_k,t_l)\,\delta_{ij}, \quad (5)$$

where $(i,s)\in\{(A,\mathrm{L}),(B,\mathrm{R})\}$ and hence s stands for the surface site of the environment. The net interaction between an electron in the system and one in the reservoir is $U_{is}=-2U^{(\mathrm{dir.})}_{is}+U^{(\mathrm{exch.})}_{is}$, where the direct term contributes with a fermion loop with an extra spin summation. Notice that the self-energy diagrams shown in Fig. 1(b) describe an electron exciting an electron–hole pair in the environment and later destroying it. The evaluation of these processes requires accounting for the different time scales for the propagation of excitations in the system and reservoirs. We resort to time-energy variables [14]: $t_i=\frac{t_k+t_l}{2}$, the physical time, and $\delta t_i=t_k-t_l$, which characterizes the quantum correlations. The integrand in Eq. (4), when $t_2=t_1=t$ becomes $\mathbf{G}^{\mathrm{R}}(t,t_i+\delta t_i/2)\mathbf{\Sigma}^{<}(\delta t_i,t_i)\mathbf{G}^{\mathrm{A}}(t_i-\delta t_i/2,t)$. Since δt_i is related to the energy ε through a Fourier transform (FT), in equilibrium,

$$\begin{aligned}G^{<}_{s,s}(\varepsilon,t_i)&=\mathrm{i}2\pi N_s(\varepsilon)\,f_s(\varepsilon,t_i),\\ G^{>}_{s,s}(\varepsilon,t_i)&=-\mathrm{i}2\pi N_s(\varepsilon)\,[1-f_s(\varepsilon,t_i)].\end{aligned} \quad (6)$$

Here $N_s(\varepsilon)$ is the local density of states (LDoS) at the surface of the reservoir and $f_s(\varepsilon,t_i)=\frac{1}{2}$ is the occupation factor in the high temperature limit ($k_BT\gg V_s$). Replacing the LDoS [15] $N_s(\varepsilon)=1/(\pi V_s)\sqrt{1-(\frac{\varepsilon}{2V_s})^2}$ in Eq. (6), and doing the FT [13] one gets $G^{\lessgtr}_{s,s}(\delta t_i,t_i)=\pm\mathrm{i}\frac{1}{V_s}\frac{J_1(\frac{2V_s}{\hbar}\delta t_i)}{\delta t_i}\frac{1}{2}$, where J_1 is the Bessel function of first order. Replacing in Eq. (5)

$$\frac{\Sigma^{\lessgtr}_{ij}(\delta t_i,t_i)}{\hbar^2}=U^2_{is}\left[\frac{J_1(\frac{2V_s}{\hbar}\delta t_i)}{2V_s\delta t_i}\right]^2 G^{\lessgtr}_{ii}(\delta t_i,t_i)\,\delta_{ij}. \quad (7)$$

$|J_1[2V_s\delta t_i/\hbar]/(V_s\delta t_i)|^2$ decays in a time scale $\hbar/V_s$ which, in the wide-band (WB) limit $V_s\gg V_{AB}$, is much shorter than $\hbar/V_{AB}$, the time scale of $G^{\lessgtr}_{ii}(\delta t_i,t_i)$. Hence, the main contribution to the integral on δt_i in Eq. (4) is obtained by replacing $G^{\lessgtr}_{ii}(\delta t_i,t_i)$ with $G^{\lessgtr}_{ii}(0,t_i)$. The same consideration holds for $\mathbf{G}^{\mathrm{R}}(t,t_i+\delta t_i/2)$ and $\mathbf{G}^{\mathrm{A}}(t_i-\delta t_i/2,t)$ which are replaced by $\mathbf{G}^{\mathrm{R}}(t,t_i)$ and $\mathbf{G}^{\mathrm{A}}(t_i,t)$. Then, the dependence on δt_i enters only through $\Sigma^{\lessgtr}_{ij}(\delta t_i,t_i)$ yielding

$$\Sigma^{\lessgtr}_{ij}(t_i)=\int\Sigma^{\lessgtr}_{ij}(\delta t_i,t_i)\,\mathrm{d}\delta t_i \quad (8)$$

$$=\frac{8}{6\pi}\frac{|U_{is}|^2}{V_s}\hbar G^{\lessgtr}_{ii}(t_i)\,\delta_{ij}. \quad (9)$$

Hence, in the WB limit, the Keldysh self-energy of Eq. (5) becomes local in space and time and no further structure from higher order terms is admissible. This is represented as a collapse of successive pairs of black dots in Fig. 1(b) into a single point. The expansion represented in Fig. 1(c) and that of Fig. 1(d) become exact in this limit.

We assume $E_A=E_B=E_{\mathrm{L}}=E_{\mathrm{R}}=0$ and the symmetry condition $|U_{A\mathrm{L}}|^2/V_{\mathrm{L}}=|U_{B\mathrm{R}}|^2/V_{\mathrm{R}}$. From Eq. (9) we obtain the decay rates

$$\begin{aligned}\frac{1}{\tau_{\mathrm{SE}}}&\equiv\frac{2}{\hbar}\Gamma_{\mathrm{SE}}\equiv\frac{-2}{\hbar}\mathrm{Im}\,\Sigma^{R}_{ii}\equiv\frac{\mathrm{i}}{\hbar}\left(\Sigma^{A}_{ii}-\Sigma^{R}_{ii}\right)\\ &=\frac{2\pi}{\hbar}|U_{is}|^2\frac{2}{3\pi^2V_s},\end{aligned} \quad (10)$$

coinciding with the FGR. The WB limit can be relaxed because the FGR holds when time t is in the range [13] $t_S<t<t_R\simeq\alpha\hbar/\Gamma_{\mathrm{SE}}\ln(V_s/\Gamma_{\mathrm{SE}})$, where α depends on the van Hove singularities of the spectral density $\mathcal{J}_s(\varepsilon)=\frac{1}{4}\int N_s(\varepsilon-\varepsilon')N_s(\varepsilon')\mathrm{d}\varepsilon'$. Here, $t_S=\hbar\mathcal{J}_s(0)\simeq\hbar/V_s$ is the survival time of an electron–hole excitation at the surface site and t_R characterizes the transition to a power law decay arising from memory effects. Hence, as long as $\Gamma_{\mathrm{SE}},V_{AB}\ll V_s$, the FGR is valid for times much longer than $\hbar/\Gamma_{\mathrm{SE}}$. Under these conditions, $\mathbf{H}_{\mathrm{eff.}}$ does not depend on ε and the propagator has a simple dependence on t as $\mathbf{G}^{\mathrm{R}}(t)=\mathbf{G}^{0\mathrm{R}}(t)\mathrm{e}^{-\Gamma_{\mathrm{SE}}t/\hbar}$, where, $G^{0\mathrm{R}}_{AA}(t)=G^{0\mathrm{R}}_{BB}(t)=\frac{\mathrm{i}}{\hbar}\cos(\frac{\omega_0}{2}t)$ and $G^{0\mathrm{R}}_{AB}(t)=G^{0\mathrm{R}}_{BA}(t)=\frac{\mathrm{i}}{\hbar}\sin(\frac{\omega_0}{2}t)$. Eq. (4) becomes,

$$\begin{aligned}\mathbf{G}^{<}(t,t)&=\hbar^2\,\mathbf{G}^{0\mathrm{R}}(t)\,\mathbf{G}^{<}(0,0)\,\mathbf{G}^{0\mathrm{A}}(-t)\,\mathrm{e}^{-t/\tau_{\mathrm{SE}}}\\ &+\int_0^t\mathrm{d}t_i\mathbf{G}^{0\mathrm{R}}(t-t_i)\,\mathbf{\Sigma}^{<}(t_i)\,\mathbf{G}^{0\mathrm{A}}(t_i-t)\,\mathrm{e}^{-(t-t_i)/\tau_{\mathrm{SE}}},\end{aligned} \quad (11)$$

a generalized Landauer-Büttiker equation [14,16]. The initial condition has the state A occupied: $\frac{\hbar}{\mathrm{i}}G^{<}_{ij}(0,0)=\delta_{iA}\delta_{Aj}$. Substituting Eq. (9) into Eq. (11), and identifying the interaction rate of Eq. (10), we get two coupled equations for $G^{<}_{AA}$ and $G^{<}_{BB}$

$$\begin{aligned}\frac{\hbar}{\mathrm{i}}G^{<}_{\substack{AA\\(BB)}}(t,t)&=\hbar^2\left|G^{0\mathrm{R}}_{\substack{AA\\(BA)}}(t)\right|^2\mathrm{e}^{-t/\tau_{\mathrm{SE}}}\\ &+\int\hbar^2\left|G^{0\mathrm{R}}_{\substack{AA\\(BA)}}(t-t_i)\right|^2\mathrm{e}^{-(t-t_i)/\tau_{\mathrm{SE}}}\frac{\mathrm{d}t_i}{\tau_{\mathrm{SE}}}\left[\frac{\hbar}{\mathrm{i}}G^{<}_{AA}(t_i)\right]\\ &+\int\hbar^2\left|G^{0\mathrm{R}}_{\substack{AB\\(BB)}}(t-t_i)\right|^2\mathrm{e}^{-(t-t_i)/\tau_{\mathrm{SE}}}\frac{\mathrm{d}t_i}{\tau_{\mathrm{SE}}}\left[\frac{\hbar}{\mathrm{i}}G^{<}_{BB}(t_i)\right].\end{aligned} \quad (12)$$

The first term in the right hand side is the probability that a particle initially at site A is found in site A (or B) at time t having survived the interactions with the environment. The second and third terms describe particles whose last interaction with the environment, at time t_i, occured at site A and B respectively. The Laplace transform yields:

$$\frac{\hbar}{\mathrm{i}}G^{<}_{AA}(t,t)=\frac{1}{2}+a_0\cos[(\omega+\mathrm{i}\eta)t-\phi]\,\mathrm{e}^{-t/(2\tau_{\mathrm{SE}})}, \quad (13)$$

where $a_0^2 = (4\omega^2\tau_{SE}^2 + 1)/(16\omega^2\tau_{SE}^2)$; $\phi = \arctan[1/(2\omega\tau_{SE})]$ and

$$\omega = \begin{cases} \omega_0\sqrt{1-(2\omega_0\tau_{SE})^{-2}} & \omega_0 > \dfrac{1}{2\tau_{SE}} \\ 0 & \omega_0 \le \dfrac{1}{2\tau_{SE}}, \end{cases} \tag{14}$$

$$\eta = \begin{cases} 0 & \omega_0 > \dfrac{1}{2\tau_{SE}} \\ \omega_0\sqrt{(2\omega_0\tau_{SE})^{-2}-1} & \omega_0 \le \dfrac{1}{2\tau_{SE}}. \end{cases} \tag{15}$$

Noticeably, in the first term of Eq. (11) the environment, though giving the exponential decay, does not affect the frequency. Modification of ω requires dynamical feedback.

The effect of lateral chains on the two state system can produce observables with non-linear dependences on H_{SE} which could account for a crossover among the limiting dynamical regimes. However, we find a *non-analyticity* in these functions enabled by the infinite degrees of freedom of the environment [17] (i.e. the thermodynamic limit). Here, they are incorporated through the imaginary part of the self-energy, $\hbar/\tau_{SE}$, i.e. the FGR. Hence, the non-analyticity of ω and τ_ϕ on the control parameter $\omega_0\tau_{SE}$ at the critical value, indicates a switch between two dynamical regimes which we call a Quantum Dynamical Phase Transition.

In the *swapping phase* the observed frequency ω is finite. According to Eq. (14), if $\omega_0\tau_{SE} \gg 1$ it coincides with ω_0, indicating a weakly perturbed evolution. As one approaches the critical value $\omega_0\tau_{SE} = \frac{1}{2}$, ω decreases, vanishing at the critical point. Beyond that value lies the *Zeno phase* where the swapping freezes ($\omega = 0$).

The "decoherence" rate observed from the attenuation of the oscillation is:

$$1/\tau_\phi = 1/(2\tau_{SE}) \quad \text{for } \omega_0 \ge 1/(2\tau_{SE}). \tag{16}$$

The dependence of the first term in Eq. (11) on $1/\tau_{SE}$ describes the decay of the initial state into system–environment superpositions. This decay is instantaneously compensated by the "reinjection" term which being "in-phase" with the Rabi oscillation ensures $1/\tau_\phi \le 1/\tau_{SE}$. Beyond the critical value, $\omega_0\tau_{SE} \le 1/2$, the decay rate in Eq. (13) *bifurcates* in *two* damping modes $1/(2\tau_{SE}) \pm \eta$. The slowest, for $\omega_0 \le \frac{1}{2\tau_{SE}}$, is:

$$1/\tau_\phi = \frac{1}{2\tau_{SE}}\left[1-\sqrt{1-(2\omega_0\tau_{SE})^2}\right] \xrightarrow[\omega_0\tau_{SE}\to 0]{} \omega_0^2\tau_{SE}. \tag{17}$$

This manifests the Quantum Zeno Effect: the stronger the interaction with the environment, the longer the survival of the initial state. The critical behavior of the observables ω and $1/\tau_\phi$ is shown in Fig. 2(a) and (b).

In Fig. 3 different colors label excess or defect in the occupation of state A with respect to equilibrium. The hyperbolic stripes show that the swapping period $T = 2\pi/\omega$ diverges beyond a finite critical value $\omega_0\tau_{SE} = 1/2$, evidencing the *dynamical phase transition*. Near the critical point $T \simeq (T_c/\sqrt{2})(1 - T_0/T_0^c)^{-1/2}$, where $T_0 = \frac{2\pi}{\omega_0}$ and $T_0^c = 4\pi\tau_{SE}$ is the critical natural period. The critical exponent is $-1/2$.

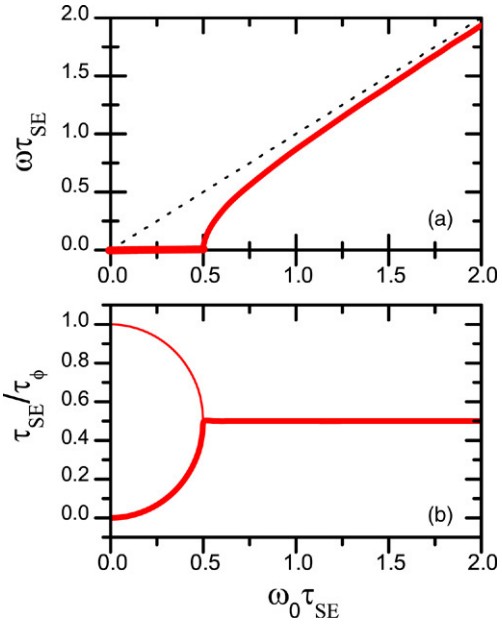

Fig. 2. (Color online) (a) Swapping frequency as a function of the control parameter $\omega_0\tau_{SE}$. Departure from the dashed line is a consequence of the environment. (b) Decoherence rate as a function of $\omega_0\tau_{SE}$. At $\omega_0\tau_{SE} = 1/2$ there is a bifurcation into two decay modes.

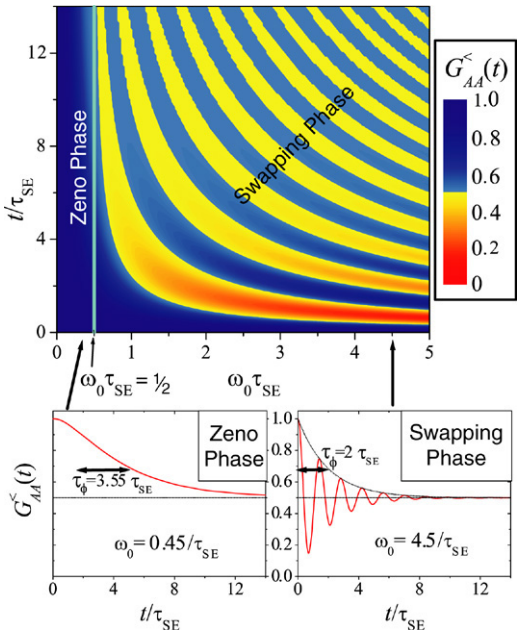

Fig. 3. (Color online) $G^<_{AA}(t)$ as a function of time and ω_0. The vertical line indicates the critical value where the oscillation period diverges. Bottom panels show $G^<_{AA}(t)$ for values of $\omega_0\tau_{SE}$ within the Zeno phase (left panel) and the swapping phase (right panel).

Typical dynamics illustrating both phases are shown at the bottom. Both start with a quadratic decay which is beyond the FGR and results from the phase ϕ in Eq. (13).

In summary, we have considered a microscopic model for an electron in a two state system coupled to an environment through a many-body interaction in the high temperature approximation. We showed that by sweeping $\omega_0\tau_{SE}$ below the critical value $\frac{1}{2}$ the oscillatory dynamics freezes in full

consistency with the experimental observation [4]. While a crossover to a Zeno regime is expected for strong SE interaction, the thermodynamic limit in the environment size gives a non-analytic point. This critical point separates a finite parametric region of dynamical freezing from a swapping phase that defines a *quantum dynamical phase transition*.

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